Variable Selection under Logistic Regression for Compositional Functional Data

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2. Model setup
3. An iterative algorithm
4. Theoretical properties
5. Simulations
6. Real world application

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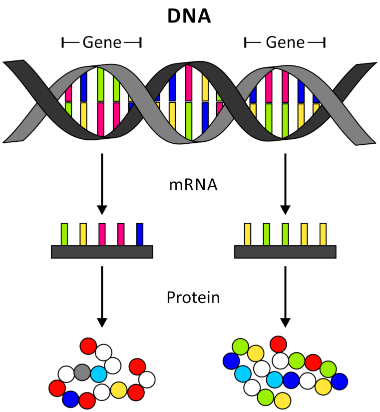
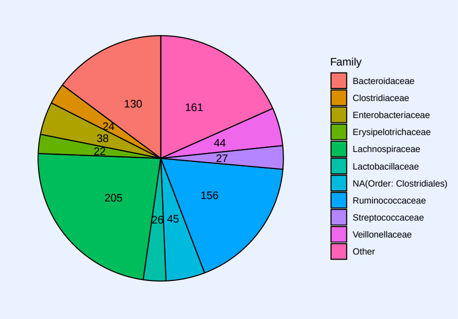
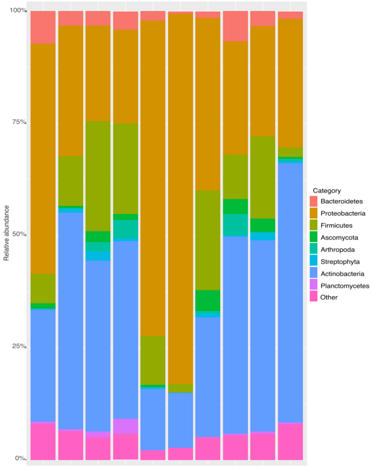
Background


DNA
Gene
Gene
mRNA
Protein


130
161
24
38
22
44
27
205
156
26
45
Family
Bacteroidaceae
Clostridiaceae
Enterobacteriaceae
Erysipelotrichaceae
Lachnospiraceae
Lactobacillaceae
NA(Order: Clostridiales)
Ruminococcaceae
Streptococcaceae
Veillonellaceae
Other

Background



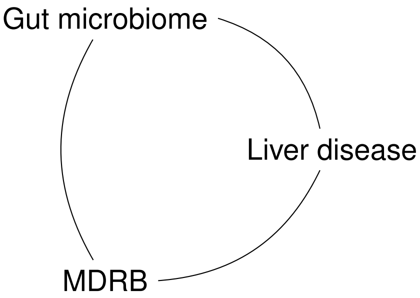
- Sequencing approaches identify numerous microbes.
- The compositional data is more meaningful than raw counts data when studying microbiome.
- Multiple sampling during the study interval naturally results in the formation of functional curves.

Background

Complexity of the data

- High-dimensional data in needs of variable selection
- Complex formation: functional and compositional

Background



- Intestinal microbiome is related to human health in many ways.
- Infections by Multidrug-resistant bacteria (MDRB) remains a leading cause of morbidity and mortality after liver transplantation.

Related work (compositional functional data analysis)

- Log-contrast model: Aitchison (1984); Lin (2014)
- Variable selection for functional data: Ramsay (2002); Fan (2015)
- Compositional functional data analysis via linear regression: Sun (2020)

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Model setup I

Log-contrast model

For compositional data $z_j \in \mathcal{R}_+, j = 1, \cdots, p$ with $\sum_j z_j = 1$, the log-contrast model proposed by (Aitchison , 1984):

$$y = \sum_{j=1}^{p-1} \beta_j \log\left(z_j / z_p\right) + \varepsilon.$$

By introducing $\beta_p = -\sum_{j=1}^{p-1} \beta_j$, the model becomes (Lin , 2014):

$$y = \sum_{j=1}^{p} \beta_j \log z_j + \varepsilon, \quad \text{s.t.} \sum_{j=1}^{p} \beta_j = 0.$$

Model setup II

Logistic model with compositional functional covariates

For binary response $y_i \in \{0, 1\}$, the logistic model for its conditional probability $\pi_i = P\left(y_i = 1 \middle| \boldsymbol{w}_i, X_{ij}(t)\right)$ is

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \alpha_0 + \boldsymbol{w}_i^T \boldsymbol{\delta}_0 + \sum_{j=1}^{p} \int X_{ij}(t)\, \beta_j(t)\, \mathrm{d}t$$

$$\text{s.t.} \sum_{j=1}^{p} \beta_j(t) = 0, \quad \forall t \in [0, 1],$$

where $X_{ij}(t) = \log Z_{ij}(t)$ is the logrithm of the compositional functional data.

Model setup III

Variable selection for functional covariates

$$\sum_{j=1}^{p} P_\lambda\left(\left\|\beta_j\left(\cdot\right)\right\|_2\right),$$

where $\left\|\beta_j\left(\cdot\right)\right\|_2 = \sqrt{\int \beta_j^2\left(t\right)\mathrm{d}t}$ represents L_2-norm of $\beta_j\left(\cdot\right)$.

Model setup IV

Low-rank approximation

Denote $\{b_k(t), k = 1, 2, \cdots\}$ a class of orthornormal basis function on $L^2(\mathrm{d}t)$. Then we have the representation:

$$X_{ij}(t) = \sum_{k=1}^{\infty} \theta_{ijk} b_k(t), \quad \beta_j(t) = \sum_{k=1}^{\infty} \eta_{jk} b_k(t).$$

Apply low-rank approximation by letting $\boldsymbol{\theta}_{ij} = (\theta_{ij1}, \cdots, \theta_{ijk_n})^T$ and $\boldsymbol{\eta}_j = (\eta_{j1}, \cdots, \eta_{jk_n})^T$. Then

$$\int X_{ij}(t) \beta_j(t) \,\mathrm{d}t \approx \boldsymbol{\theta}_{ij}^T \boldsymbol{\eta}_j, \quad \text{and} \quad \|\beta_j(t)\|_2 \approx \|\boldsymbol{\eta}_j\|_2,$$

where $\|\boldsymbol{\eta}_j\|_2 = \sqrt{\boldsymbol{\eta}_j^T \boldsymbol{\eta}_j}$ represents the L_2-norm of a vector.

Model setup V

Low-rank representation of the original model

$$\min S_{\lambda}^{\star}(\beta) = -\frac{1}{n} L(\beta) + \sum_{j=1}^{p} P_{\lambda}\left(\|\eta_j\|_2\right)$$

$$\text{s.t.} \sum_{j=1}^{p} \eta_j = \mathbf{0}_{k_n},$$

where $L(\beta)$ is the MLE of the data, $\beta = \left(\alpha, \delta^T, \eta_1^T, \cdots, \eta_p^T\right)^T$ is the vector of unknown parameters.

Model setup VI

Augmented Lagrangian Multiplier(ALM) method

$$S_\lambda(\beta) = -\frac{1}{n}L(\beta) + \sum_{j=1}^{p} P_\lambda\left(\|\eta_j\|_2\right) + \boldsymbol{\mu}_1^T \sum_{j=1}^{p} \boldsymbol{\eta}_j + \frac{\mu_2}{2}\left\|\sum_{j=1}^{p} \boldsymbol{\eta}_j\right\|_2^2,$$

where $\boldsymbol{\mu}_1$ is the multiplier vector and $\mu_2 > 0$ is the parameter for the augmented term.

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An iterative algorithm

Quadratic approximation at β_0

$$-\frac{1}{n}L(\beta) \approx \frac{1}{n}\left(-L(\beta_0) + (\beta - \beta_0)^T \boldsymbol{X}^T (\pi - \boldsymbol{Y}) + \frac{1}{2}(\beta - \beta_0)^T \boldsymbol{X}^T \boldsymbol{W}\boldsymbol{X}(\beta - \beta_0)\right),$$

where $\boldsymbol{W} = \mathrm{diag}(\pi_i(1 - \pi_i)) \leq \frac{1}{4}\boldsymbol{I}_n$.

Quadratic Majorization

Replace $\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{X}$ with $\boldsymbol{H} = \frac{1}{4}\boldsymbol{X}^T\boldsymbol{X}$.

An iterative algorithm (updating η_{j_0}) I

Apply MM-principle multiple times

$$
\begin{aligned}
S_\lambda\left(\eta_{j_0}\right) \leq\; & \frac{1}{2}\eta_{j_0}^T\left(\frac{d^2_{j_0,max}}{4n}\boldsymbol{I}_{k_n} + \mu_2\boldsymbol{I}_{k_n}\right)\eta_{j_0} \\
& - \eta_{j_0}^T\left(\frac{d^2_{j_0,max}}{4n}\eta_{j_0}^{(m)} - \frac{1}{n}\Theta_{j_0}^T\left(\boldsymbol{\pi}^{(m)} - \boldsymbol{Y}\right) - \boldsymbol{\mu}_1^{(m)} - \mu_2\sum_{j\neq j_0}\eta_j^{(m)}\right) \\
& + P_\lambda\left(\left\|\eta_{j_0}\right\|\right),
\end{aligned}
$$

where $d^2_{j_0,max}$ is the maximum eigen value of $\Theta_{j_0}^T\Theta_{j_0}$ and Θ_{j_0} is the low-rank representation of the j_0th functional covariate.

An iterative algorithm (updating η_{j_0}) II

Local Linear Approximation(LLA)

$$P_\lambda\left(\|\eta_{j_0}\|\right) \approx P_\lambda\left(\left\|\eta_{j_0}^{(m)}\right\|\right) + P'_\lambda\left(\left\|\eta_{j_0}^{(m)}\right\|\right)\left(\|\eta_{j_0}\|_2 - \left\|\eta_{j_0}^{(m)}\right\|_2\right).$$

Updating step for η_{j_0}

$$\eta_{j_0}^{(m+1)} = \frac{1}{d_{j_0,\max}^2/(4n) + \mu_2}\left(1 - \frac{P'_\lambda\left(\left\|\eta_{j_0}^{(m)}\right\|\right)}{\sqrt{\alpha_{j_0}^{(m)T}\alpha_{j_0}^{(m)}}}\right)_+ \alpha_{j_0}^{(m)},$$

where

$$\alpha_{j_0}^{(m)} = \frac{d_{j_0,\max}^2}{4n}\eta_{j_0}^{(m)} - \frac{1}{n}\Theta_{j_0}^T\left(\pi^{(m)} - Y\right) - \mu_1^{(m)} - \mu_2\sum_{j\neq j_0}\eta_j^{(m)}.$$

Implementations in R package I

Sample weighting

$$L_v = \sum_{i=1}^{n} v_i \left(y_i \pi_i + (1 - y_i)(1 - \pi_i)\right),$$

where $v_i, i = 1, \cdots, n$ is a set of positive weights.

Implementations in R package II

Flexibility and robustness

within-group standardization

multiplier vector

$$S_n = \frac{-1}{a} L_v + \sum_{j=1}^{p} c_j P_{\lambda_n}\left(b_j \|\boldsymbol{\eta}\|_2\right) + \boldsymbol{\mu}_1^T \sum_{j=1}^{p} \boldsymbol{\eta}_j + \frac{\mu_2}{2} \left\| \sum_{j=1}^{p} \boldsymbol{\eta}_j \right\|_2^2 + \sum_{j=1}^{h+pk_n} \frac{r_j}{2} \beta_j^2,$$

weights for different modules

robust term

Implementations in R package III

Within-group orthonormalization

$$S_n = \frac{-1}{a} L_v + \sum_{j=1}^{P} c_j P_{\lambda_n}\left(b_j \left\|\tilde{\eta}_j\right\|_2\right) + \boldsymbol{\mu}_1^T \sum_{j=1}^{P} \boldsymbol{T}_j \tilde{\boldsymbol{\eta}}_j + \frac{\mu_2}{2} \left\|\boldsymbol{T}_j \tilde{\boldsymbol{\eta}}_j\right\|_2^2 + \sum_{j=1}^{h+\sum_k m_k} \frac{r_j}{2} \beta_j^2,$$

where $\tilde{\eta}_j$ is the parameters after within-group orthonormalization. $\boldsymbol{T}_j = \sqrt{a} \boldsymbol{V}_j \boldsymbol{D}_j^{-1}$ is the transforming matrix satisfying $\eta_j = T_j \tilde{\eta}_j$. Covariates after transforming $\tilde{\boldsymbol{\Theta}}_j = \boldsymbol{\Theta}_j \boldsymbol{T}_j$ satisfies $\frac{1}{a} \tilde{\boldsymbol{\Theta}}_j^T \tilde{\boldsymbol{\Theta}}_j = \boldsymbol{I}$.

Implementations in R package IV

FLiRTI(James , 2009) procedure

Pursuit the functional parameters in simple form of curves:

$$S_n = \text{Logistic} + \sum_{j=1}^{q}\sum_{k=1}^{T} c_{jk} P_{\lambda}\left(b_{jk}\gamma_{(1),jk}\right) + \boldsymbol{\mu}_1^T \sum_{j=1}^{q}\gamma_{(1),j} + \frac{\mu_2}{2}\left\|\sum_{j=1}^{q}\gamma_{(1),j}\right\|_2^2 .$$

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Convergence of the algorithm

Checking KKT conditions we know that

Theorem

The objective function coupled with Lasso penalty strictly descents during the iteration. If the current solution $\alpha^{(m+1)}$, $\delta^{(m+1)}$, $\eta_j^{(m+1)}, j = 1, \cdots, p$ and $\mu_1^{(m+1)}$ stay unchanged after one complete iteration, then the algorithm reaches the minimum point of the objective function.

Consistency of the Lasso estimator I

Theorem (Consistency of the Lasso estimator)

If Assumption 1–3 hold and $pk_n = o\left(e^n\right)$, $\lambda \to 0$. Also there exists constant c such that $\lambda > c\sqrt{k_n \log\left(pk_n\right)/n}$. Then with probability approaching to 1, the Lasso estimator $\hat{\xi}^{glasso}$ satisfies

$$\left\|\hat{\xi}^{glasso} - \xi^\star\right\|_2 \leq 2\left(1+\phi\right)\left(2+\phi\right)\frac{\sqrt{q_n}\lambda}{\kappa},$$

as $n \to \infty$, where ϕ is any positive constant and $\xi^\star$ is the real underlying parameters.

Consistency of the Lasso estimator II

Corollary

Under the condition that the previous theorem holds, and additionally the minimal signal strength Assumption 4 holds, the with probability approaching to 1,

$$\min_{j\in\{1,\cdots,q_n\}} \left\|\hat{\eta}_j^{glasso}\right\|_2 \geq 2\left(1+\phi\right)\left(2+\phi\right)\frac{\sqrt{q_n}\lambda}{\kappa},$$

and

$$\max_{j\in\{q_n+1,\cdots,p\}} \left\|\hat{\eta}_j^{glasso}\right\|_2 \leq 2\left(1+\phi\right)\left(2+\phi\right)\frac{\sqrt{q_n}\lambda}{\kappa}.$$

as $n \to \infty$.

Necessary assumptions I

1. There uniformly exist positive constants K and R, independent of n, such that for $1 \leq i \leq n$, $1 \leq j \leq p$, $1 \leq k \leq k_n$ and $m = 2, 3, \cdots,$

$$\mathrm{E}\left|\theta_{ij,k}\left(y_i - \pi_i\left(\xi^\star, \boldsymbol{\Theta}\right)\right)\right|^m \leq (m!/2)\, K^{m-2} R^2,$$

   where we use $\pi_i\left(\xi^\star, \boldsymbol{\Theta}\right)$ to emphasize that the computation of π_i relies on underlying parameters $\xi^\star$ and data $\boldsymbol{\Theta}$.

2. There exists a positive constant κ such that

$$\inf\left\{\frac{\sqrt{\Delta^T \boldsymbol{\Theta}^T \boldsymbol{W}\left(\xi^\star, \boldsymbol{\Theta}\right) \boldsymbol{\Theta}\Delta}}{\sqrt{n}\,\|\Delta\|_2} : \boldsymbol{C}^T\Delta = \mathbf{0}, \sum_{j=q_n+1}^{p} \|\Delta_j\|_2 \leq (1+\phi) \sum_{j=1}^{q_n} \|\Delta_j\|_2\right\} = \kappa > 0,$$

   where $\boldsymbol{\Theta} = \left(\boldsymbol{\Theta}_1, \cdots, \boldsymbol{\Theta}_p\right)$ is the $n \times (pk_n)$ low-rank representation of p functional covariates.

Necessary assumptions II

3. $\mathrm{E}q_3(\xi, \theta)\|\theta\|_{2,\infty}^3$ is bounded in the neighbourhood centered at $\xi^\star$, where $q_3(\xi, \theta)$ represents the 3rd order derivative of the link function of logistic regression.

4. The real underlying model parameters satisfy

$$\min_{j\in\{1,\cdots,q_n\}} \left\|\eta_j^\star\right\|_2 \geq 4(1+\phi)(2+\phi)\sqrt{q_n}\lambda.$$

Oracle property when applying non-convex penalty

Theorem

If Assumption 1–3, 5–7 hold, and $\lambda = o\left(n^{-(1-c_2)/2}\right)$, $q_n k_n \sqrt{k_n/n} = o(\lambda)$, $\sqrt{k_n}\log p = o(n\lambda)$ *and* $n\lambda/\sqrt{k_n} \to \infty$ *are satisfied, then there exists a local minimum* $\left(\hat{\eta}_1^T, \cdots, \hat{\eta}_p^T\right)^T$ *of the objective function coupled with SCAD or MCP penalty such that*

$$P\left(\left(\hat{\eta}_1^T, \cdots, \hat{\eta}_p^T\right)^T = \left(\hat{\eta}_1^{or\ T}, \cdots, \hat{\eta}_p^{or\ T}\right)^T\right) \to 1$$

as $n \to \infty$, *where* $\left(\hat{\eta}_1^{or\ T}, \cdots, \hat{\eta}_p^{or\ T}\right)^T$ *denotes the oracle estimator.*

Necessary assumptions

5. There exist two positive constants C_1 and C_2 uniformly for $j \in \{1, \cdots, q_n\}$, such that

$$0 < C_1 \leq \lambda_{min}\left(\frac{1}{n}\Theta_j^T \boldsymbol{W}\left(\xi^\star, \Theta\right)\Theta_j\right) \leq \lambda_{max}\left(\frac{1}{n}\Theta_j^T \boldsymbol{W}\left(\xi^\star, \Theta\right)\Theta_j\right) \leq C_2,$$

   where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ represents the minimum and maximum eigen value of a given matrix. Also it's assumed that $\max_{1\leq i\leq n}\left\|\left(\boldsymbol{\theta}_{i1}^T, \cdots, \boldsymbol{\theta}_{iq_n}^T\right)\right\|_2 = O_p\left(\sqrt{q_n k_n}\right)$ and there exists a constant M_1 such that $\max_{j,k}\mathrm{E}\left|\theta_{\cdot jk}\right| \leq M_1$ for $j \in \{q_n + 1, \cdots, p\}$, $k \in \{1, \cdots, k_n\}$.
6. There exists a positive constant c_1 such that $0 \leq c_1 < \frac{1}{3}$ and $q_n k_n = O\left(n^{c_1}\right)$.
7. There exists a positive constant c_2 such that $2c_1 < c_2 < 1$ and

$$n^{(1-c_2)/2}\min_{1\leq j\leq q_n}\left\|\boldsymbol{\eta}_j^\star\right\|_2 \geq M_2.$$

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Simulation settings I

Data generation

$$\bar{W}_{ij}(t) = \exp\left(\boldsymbol{w}_{ij}^{T}\boldsymbol{c}(t) + \varepsilon_{ij}(t)\right), \quad i = 1, \cdots, n, \quad j = 1, \cdots, p,$$

where $\boldsymbol{c}(t)$ is a set of Fourier basis, $\varepsilon_{ij}(t)$ follows $N(0, 0.5^2)$ at each time points. The functional counting data is acquired by $W_{ij}(t) = \lfloor \bar{W}_{ij}(t) \rfloor$. Hence

$$X_{ij}(t) = \log\left(\frac{W_{ij}(t) + 1}{\sum_{j=1}^{p} W_{ij}(t) + p}\right), \quad i = 1, \cdots, n, \quad j = 1, \cdots, p.$$

Simulation settings II

Low-rank approximation

- First q_n terms of $\beta_j(t)$ are generated from $\beta_j = \boldsymbol{\eta}_j^T \boldsymbol{c}(t)$, where $\boldsymbol{\eta}_j$ is centerd so that $\sum_{j=1}^{q_n} \boldsymbol{\eta}_j = \mathbf{0}$. The remaining $\beta_j(t), j = q_n + 1, \cdots, p$ are set to constant 0.
- Orthonormalized B-splineRedd (2011) is applied for low-rank approximation.

Magnitude of the simulation

- Sample size $n \in \{75, 100, 150, 250\}$, number of functional covariates $p \in \{50, 500\}$.
- Number of important covariates $q_n \in \{8, 10\}$, number of basis functions $k_n = 6$.

Simulation settings III

Candidate methods

- Oracle model
- Proposed model, coupled with MCP and Lasso penalty
- GGL (Generalized linear regression with Group Lasso, Yang (2014))
- ZINB (Zero Inflated Negative Binomial model, Zhang (2018))

Simulation results (low-dimensional settings) I

Table 1: $n = 150$, $p = 50$, $q = 10$

Model	Criteria	AUC	MSE_δ	$\lvert\sum\hat{\eta}\rvert_\infty$	FP	FN	FDR
Oracle	-	0.947(0.039)	0.711(0.439)	$< 10^{-5}$	0(0)	0(0)	0(0)
MCP	BIC	0.881(0.081)	0.847(0.477)	$< 10^{-5}$	1.010(0.969)	3.860(1.059)	0.138(0.131)
MCP	CV	0.827(0.095)	0.794(0.272)	$< 10^{-5}$	0.150(0.411)	6.420(0.987)	0.035(0.093)
Lasso	BIC	0.870(0.070)	0.803(0.456)	$< 10^{-5}$	1.410(1.156)	3.840(0.801)	0.172(0.130)
Lasso	CV	0.843(0.075)	0.766(0.250)	$< 10^{-5}$	18.030(3.465)	0.790(0.715)	0.657(0.046)
GGL	CV	0.877(0.069)	2.046(1.977)	0.546	12.510(6.522)	1.380(1.277)	0.538(0.172)
ZINB	fdr0.05	0.599(0.092)	1.208(1.255)	0.107	0(0)	9.810(0.465)	0(0)

Simulation results (low-dimensional settings) II

Table 2: $n = 250$, $p = 50$, $q = 10$

Model	Criteria	AUC	MSE_δ	$\lvert\sum\hat{\eta}\rvert_\infty$	FP	FN	FDR
Oracle	-	0.973(0.020)	0.411(0.262)	$< 10^{-5}$	0(0)	0(0)	0(0)
MCP	BIC	0.954(0.030)	0.483(0.204)	$< 10^{-5}$	0.350(0.672)	1.950(0.770)	0.040(0.076)
MCP	CV	0.927(0.052)	0.587(0.225)	$< 10^{-5}$	0.150(0.557)	3.670(1.596)	0.016(0.054)
Lasso	BIC	0.949(0.032)	0.503(0.194)	$< 10^{-5}$	0.700(0.847)	2.230(0.777)	0.074(0.086)
Lasso	CV	0.884(0.053)	0.617(0.167)	$< 10^{-5}$	23.750(3.020)	0.220(0.416)	0.706(0.031)
GGL	CV	0.934(0.037)	1.505(1.330)	0.472	14.850(7.124)	0.590(0.698)	0.571(0.145)
ZINB	fdr0.05	0.573(0.076)	1.111(0.943)	0.078	0.010(0.010)	9.880(0.327)	0.010(0.100)

Simulation results (high-dimensional settings) I

Table 3: $n = 150$, $p = 500$, $q = 8$

Model	Criteria	AUC	MSE_δ	$\lvert\sum\hat{\eta}\rvert_\infty$	FP	FN	FDR
Oracle	-	0.947(0.031)	1.093(0.704)	$< 10^{-5}$	0(0)	0(0)	0(0)
MCP	BIC	0.803(0.100)	0.769(0.516)	$< 10^{-5}$	2.920(1.839)	3.200(1.325)	0.362(0.209)
MCP	CV	0.738(0.098)	0.518(0.214)	$< 10^{-5}$	0.580(0.673)	5.280(1.230)	0.150(0.168)
Lasso	BIC	0.804(0.091)	0.587(0.453)	$< 10^{-5}$	3.615(1.670)	2.769(1.231)	0.398(0.157)
Lasso	CV	0.760(0.061)	0.466(0.175)	$< 10^{-5}$	31.173(8.740)	0.981(1.180)	0.804(0.059)
GGL	CV	0.794(0.093)	3.727(4.126)	0.440	26.580(16.519)	1.460(1.854)	0.704(0.225)
ZINB	fdr0.05	0.571(0.061)	0.925(1.330)	0.033	4.260(2.039)	7.900(0.364)	0.966(0.149)

Simulation results (high-dimensional settings) II

Table 4: $n = 250$, $p = 500$, $q = 8$

Model	Criteria	AUC	MSE_δ	$\lvert\sum\hat{\eta}\rvert_\infty$	FP	FN	FDR
Oracle	-	0.954(0.023)	1.892(0.947)	$< 10^{-5}$	0(0)	0(0)	0(0)
MCP	BIC	0.951(0.047)	0.233(0.211)	$< 10^{-5}$	0.750(1.065)	0.062(0.250)	0.077(0.101)
MCP	CV	0.929(0.057)	0.185(0.127)	$< 10^{-5}$	0.312(0.602)	0.938(1.482)	0.033(0.063)
Lasso	BIC	0.937(0.036)	0.183(0.127)	$< 10^{-5}$	2.111(2.147)	0.444(0.726)	0.194(0.150)
Lasso	CV	0.840(0.068)	0.314(0.111)	$< 10^{-5}$	39.444(2.963)	0.000(0.000)	0.831(0.011)
GGL	CV	0.921(0.045)	1.442(1.232)	0.322	28.000(19.280)	0.062(0.250)	0.707(0.185)
ZINB	fdr0.05	0.590(0.076)	0.605(0.127)	0.023	5.438(2.449)	7.938(0.250)	0.992(0.031)

Simulation results (settings that mimic real data)

Table 5: $n = 150$, $p = 500$, $q = 8$. Subjects are heterogeneous, mimicing the pattern in real data application.

Model	Criteria	AUC	MSE$_\delta$	$\lvert\sum \hat{\eta}\rvert_\infty$	FP	FN	FDR
Oracle	-	0.883(0.055)	2.080(2.588)	$< 10^{-5}$	0(0)	0(0)	0(0)
MCP	BIC	0.797(0.081)	0.630(0.837)	$< 10^{-5}$	5.000(1.732)	5.348(0.775)	0.637(0.135)
MCP	CV	0.797(0.085)	0.782(0.551)	$< 10^{-5}$	1.565(2.063)	6.391(1.438)	0.257(0.283)
Lasso	BIC	0.908(0.026)	0.287(0.130)	$< 10^{-5}$	2.000(1.000)	5.200(0.837)	0.410(0.175)
Lasso	CV	0.780(0.047)	1.664(0.742)	$< 10^{-5}$	20.400(9.764)	3.800(1.304)	0.811(0.063)
GGL	CV	0.815(0.070)	1.604(1.306)	0.758	18.913(16.673)	4.565(1.343)	0.737(0.216)
ZINB	fdr0.05	0.773(0.082)	$> 4 \times 10^3$	1.326	0.522(0.790)	6.217(1.043)	0.159(0.259)

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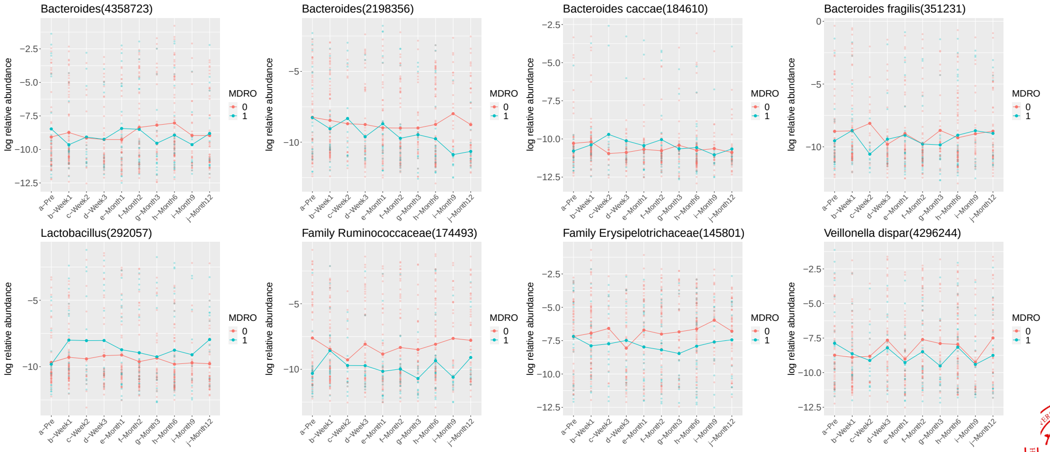
3. An iterative algorithm
4. Theoretical properties
5. Simulations
6. Real world application

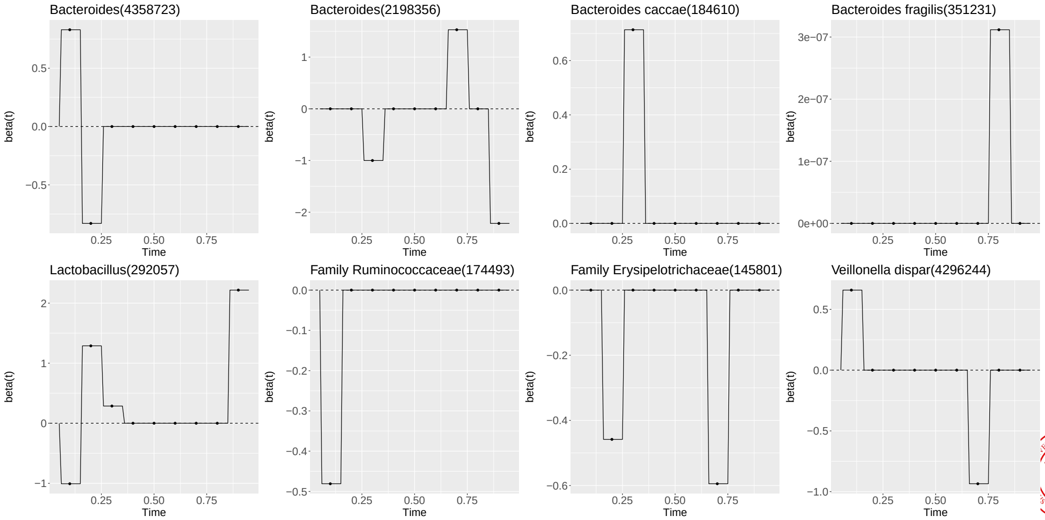
Colonizing MDRB and the intestinal microbiome

- Infections by Multidrug-resistant bacteria (MDRB) remain a leading cause of morbidity and mortality after liver transplantation(LT).
- Gut dysbiosis characteristic of end-stage liver disease may predispose patients to intestinal MDRB colonization and infectoin, in turn exacerbating dysbiosis.
- After quality control, data of 131 patients during one year after LT is collected. At Optional Taxonomic Units(OTU) level, 878 different taxons are identified.
- Colonizing status of MDRB for each patients is taken as response.

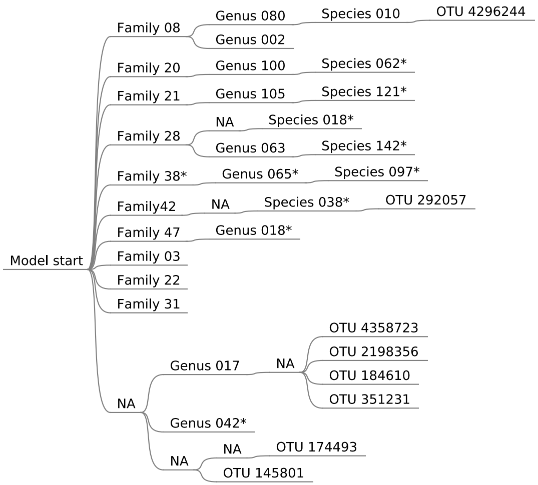
Variable selection



Estimation of functional parameters (after FLiRTI procedure)



Results at different taxonomic levels



Taxonomic levels: Family, Genus and Species.
NA means the corresponding taxon is not included in the model.

Asterisk means the current level is not officially recognized.

Future work

- Deal with confounders such as antibiotic treatment
- From observational study to causal inference
- Post-selection inference

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